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TE 855
Final Paper

Converting Between Mixed Numbers and Improper Fractions with Reason

1. Statement of the problem or question:

I am currently in my third year of teaching fifth grade. Fractions are a large component of the fifth grade curriculum. I have found that many students struggle with understanding fractions. It is essential that students have a conceptual understanding of fractions, especially since new concepts will build from these fundamental concepts taught in elementary school. When I learned about fractions, I was taught procedures. I believe many teachers teach students procedures rather than conceptual understanding since it is easier and less time consuming. Many students know or learn the “tricks” to fractional problems, but do they conceptually understand what they are doing? For instance, adding and subtracting fractions with unlike denominators, changing numbers from a mixed number to an improper fraction and vice versa. Do students have a visual of what they are doing to the fractions or why these procedures work? Knowing this information and having a better understanding will help students in the future. Therefore, I would like to investigate the question of “How do teachers help students learn to convert between mixed numbers and improper fractions with reason rather than only utilizing procedures?”

I believe if students understand how to convert between mixed numbers and improper fractions conceptually, then students will have a better understanding of how to solve the problem and will be better able to remember the concepts. From conducting research and reading research based articles, I have found that students will develop a deeper understanding of how to convert between mixed numbers and improper fractions through building with manipulatives, drawing, labeling, and discussing their reasoning.

2. A Review of Literature:

I read many articles related to my research question. The following articles helped shape my inquiry: "Reflecting on Learning Fractions without Understanding," "From the Classroom: Mixed Numbers Made Easy: Building and Converting Mixed Numbers and Improper Fractions," "Using Virtual Manipulatives to Model Computation with Fractions," and "Number, Operation, and Quantitative Reasoning." These articles support my question and gave me valuable information in how to go about teaching mixed numbers and improper fractions conceptually and with reason.

Philipp and Vincent's article is about the order in which students should learn concepts conceptually verse procedurally. Should students "learn concepts before, concurrently with, or after learning procedures"? They looked to a study done by Pesek and Kirshner in 2000. In this study, there were two groups of children. The first group was taught five days of procedures, followed by three days of conceptual learning. The other group was taught only the same three day conceptual unit. One would think the group that received eight days of instruction would perform better on an assessment than the group who only received three days of instruction. However, this study's results show that "students who received the procedural instruction prior to the conceptual instruction learned less than did students who received only the conceptual instructions." I found Rachel's, a student in the study, comment to be very interesting and informative. "She went on to explain, 'So when I figure that out, it's easier, and, um, once I figure it out, it's, it stays there 'cause I was the one who brought it there. So, and it is just easier to do when you figure it out yourself, instead of having teachers telling you.'" I believe students take ownership of their learning when they discover the concepts and develop the procedures on their own. This article does not go into detail of how to teach conceptually, however it proves that teaching conceptually is essential. I will make sure to focus on teaching my students how to convert between mixed numbers and improper fractions conceptually before discussing the procedures of how to convert between them.

Neumer's article focused on the order you must teach to build conceptual knowledge. One must first have prior knowledge of representing a whole number as a fraction by using numbers, pictures, and manipulatives. This article suggests using pattern blocks. Neumer prefers using Unifix cubes to represent mixed numbers. The students are always required to build, draw, and label their mixed numbers. Students need to have a strong understanding of the concept of mixed numbers before converting mixed numbers to improper fractions. When they are ready, students must again build, draw, and label their mixed number. Once it is drawn, students will figure out how many total cubes there are to get the numerator of the improper fraction. Students may count all the cubes or multiply the number of wholes by the number of stacked cubes and then add the additional cubes, which is the numerator of the fractional portion. This helps students visualize “where the multiplying and the adding of the algorithm occur” (p.490). Again, to convert from improper fractions to mixed numbers one must build the improper fraction, draw, and label it. Students will break the cubes into as many wholes as they can and the cubes left over is the numerator of the mixed fraction. After doing several of these problems, students will begin to realize that they are dividing and understand where the remainder is coming from. This article also believes that students need to have conceptual knowledge prior to procedural knowledge. They have the students use their conceptual knowledge to develop procedures. The students come up with the procedures after working through many conceptual examples. This helps students take ownership of their learning.

“Using Virtual Manipulatives to Model Computation with Fractions” emphasized the point that “teachers need to give students experiences in using a wide range of visual representations and introduce them to new forms of representations that are useful for solving certain types of problems” (NCTM 2000, p. 284). Virtual manipulatives are a great additional resource which allows students to become more flexible with using all three models of fractions. These models include area, length, and set. By using these three models, students will develop a deeper understanding of fractions. Students will then be able to derive their own methods and strategies for computing fractional problems. Virtual manipulatives are wonderful for teachers who have a limited variety of concrete manipulatives. In addition, “students may find working on a computer with virtual manipulatives more desirable than using concrete manipulatives that they might view as childish” (Schackow, 2006-7).

“Number, Operation, and Quantitative Reasoning” provides a variety of hands on activities and questions to ask students. It also gives a large list of manipulatives that can be used to represent length, area, and set models. The students will use these models to create procedures for converting between mixed numbers and improper fractions. Students “should be able to develop and express a generalization or rule that makes sense to them” (2006). Furthermore, I found it very valuable that students had to think about real world examples of where they see mixed numbers and improper fractions.

These articles support my research question. It is essential to teach children conceptually before teaching them procedures. All of these articles emphasize this idea. “Reflecting on Learning Fractions without Understanding” focused on the order conceptual and procedural knowledge is taught. Their study supports that students must first understand the concepts conceptually before learning the procedures. “From the Classroom: Mixed Numbers Made Easy: Building and Converting Mixed Numbers and Improper Fractions” discusses the importance of students developing an understanding of the concepts and being able to represent them before moving on. This article explains the order of developing conceptual understanding by having students build with manipulatives, draw, and label their fractions. After completing several examples, students will derive their own procedures. These articles assisted me in teaching my students how to convert between mixed numbers and improper fractions with reason rather than only utilizing procedures.

3. Modes of inquiry:

I began to answer my question by conducting research. This has provided me with strategies which support and enhance my students' conceptual understanding of converting between mixed numbers and improper fractions. First of all, I gave my students a pretest. This informed me of their level of understanding this concept before I began to teach this subject matter. I then conducted hands on activities which were based on the "Mixed Numbers Made Easy: Building and Converting Mixed Numbers and Improper Fractions" article by Chris Neumer. These activities were to lead students to a conceptual understanding. I recorded the group's discussion, made observations of students' written and verbal explanations, and collected student work. I used a Flip camera to record our sessions to better analyze my students' explanations and to determine if conceptual understanding was helpful. I focused on student explanations rather than the right or wrong answer. My goal is for students to be able to explain why they are performing different procedures. Students should be able to build with manipulatives, draw, label, and verbally explain what they are doing. Lastly, I administered a post-test after teaching these lessons. This way I was able to conclude their level of mathematical reasoning. Lastly, I compared and contrasted the students' pre and post tests to determine if the activities and strategies that I implemented with my students were effective in improving my student's mathematical reasoning of converting mixed numbers and improper fractions.

4. Results:

From analyzing the pre-test, I found that some students knew how to convert between mixed numbers and improper fractions and that they were able to explain the procedures of how to do so. However, none of the students were able to draw, label, or explain their thoughts on how and why these procures worked. As a result, these students may have had prior procedural knowledge, however they did not have conceptual knowledge.

I informed my students that we will be working with manipulatives to build, draw, and label mixed numbers and improper fractions to help us understand how to convert between the two. My students immediately asked me why we had to build, draw, and label our mixed numbers when we already know that we multiply the dominator by the whole number and then add the numerator to get the improper fraction. This proves to me that students are used to memorizing procedures and are not typically asked to explain their reasoning or give proof.

The following analysis occurred with a small group of students during lunch over a period of six days. We did many examples but I will be focusing my analysis on a few examples. The following was my teaching sequence that I presented to my students over these six days:

- Pre-test

- Representing a whole number
- Representing a mixed number
- Converting from mixed number to improper-fraction
- Representing a improper fraction
- Converting improper fractions to mixed numbers
- Post test

I began my first lesson by using pattern blocks to help build students' understanding of how to represent a whole number as a fraction. Then we moved on to representing a mixed number with unifix cubes. I had the students build 2 3/6. The students took some times to build their mixed number. I told them they could work together. A couple of students were quickly able to build 2 3/6 while the other student observed what they were doing and did the same thing. The students all built 2 towers of 6 cubes and 1 tower of 3 cubes. I wanted to ensure the students understood why this represents 2 3/6. We had the following discussion:

Mixed Numbers

Sherwood: Can anyone explain what they built?

S1: I have 2 whole numbers and 3/6. These two are the wholes (pointing to 2 of the towers that are the same height) and this is 3/6 (pointing to the smaller tower).

Sherwood: How did you know how many cubes to use to make a whole number?

S1: Always take the denominator that is always your big number.

Sherwood: What do you mean?

S1: The denominator equals number of blocks you need to make a whole number.

S2: 6 is denominator. Use as whole. I stacked 6 which equals 6/6. There are 2 wholes and 3 cubes to represent 3/6.

Sherwood: Does everyone agree with this or does someone think of it differently?

S3: 6/6 equals a whole. I agree with that, but I knew 1/6 equals 1/6 of a whole. 6 is the denominator. I would put 2 towers of 6 cubes each. Then I have a tower of 3 cubes which represents 3/6.

The students were really beginning to understand conceptually what is a mixed number. This also helped them better understand what a whole number is made of. I wanted to make sure the students understood how they created their whole number towers and were able to explain it since this would be important when converting. Students need to have a strong foundation in order to build upon these skills to understand more complicated concepts.

The students and I worked together to draw and label our mixed numbers. I then asked if anyone would like to explain their drawing to the group.

S1: I have this little sheet of paper that says 1/6, 1/6, 1/6, 1/6, 1/6, 1/6, which is 6/6, which is 1 whole and 1/6, 1/6, 1/6 which is 3/6. Then I have 1 whole plus 1 whole plus 3/6, which is 2 3/6.

My students were beginning to get comfortable explaining and talking about their work. They were excited to share what they did and why. My students were now ready to convert mixed numbers to improper fractions since they understood what the whole number and fraction represents.

Sherwood: Does anyone know how many total sixths make up this mixed number, 2 3/6. We are trying to figure out the improper fraction.

A few students: began to count all the cubes, they said 15.

Sherwood: 15, what?

All students: 15/6

Sherwood: Does anyone know and easier way of figuring out how many sixths are in this mixed number without counting?

S3: Oh I know how to do this my mom taught me. You can multiply!

Sherwood: What do you mean you can multiply? Please explain.

S3: You multiply 6x2 then add 3.

Sherwood: Why do you multiply 6x2?

S3: 6 sixths make up a whole and there are 2 wholes so you multiply 6x2. Then you have to add the fraction, which is 3/6.

S3 wrote on her paper: (6x2) + 3/6 = 12/6 + 3/6 = 15/6.

S3: I put parenthesis because you always do that first. Multiplying is a faster and easier way than counting all those sixths or cubes.

From there, we further discussed the procedures. I asked how we could solve these problems if we didn't build and draw these mixed numbers. The kids came up with the procedure of multiplying the whole number by the denominator then adding the numerator to that number. I asked why you can multiply the whole by the denominator and they responded that there are six sixths within the whole. Students were easily able to make the transition of how to convert from a mixed number to an improper fraction. These students now understand why they multiply the denominator by the whole number.

Improper Fractions

Next, I had my students build the improper fraction 14/4. My students had a very difficult time coming up with how to first make an improper fraction. I thought this would be very easy, but my assumption was wrong.

Sherwood: We are now going to practice making improper fractions. Let's start with 14/4. I want to see how you will build this.

S1: Oh, and then we are going to convert this into a mixed number!? Oh, I know how to do this! You just do the opposite, you divide and then you subtract.

Sherwood: That is a great idea, let's see if that works.

S1: I hope I got my prediction right.

Sherwood: Let’s see if S1’s prediction works and what you guys think. I want you to begin by building 14/4.

As I reflect upon by teaching practices of this research, I believe there are areas I could have improved upon. For instance, I would have asked the students how we could have tested S1 prediction instead of directing the students with my own agenda. As a teacher, it is important to listen to the students’ ideas and guide the discussion based on their thinking. I directed the students in the direction I wanted them to go verses letting the students lead their own discussion.

S1: Oh I know how to do this. S1 built 3 2/4.

Sherwood: That does represent that fraction, but what type of number did S1 build?

Students: A mixed number.

Sherwood: How can we represent it as an improper fraction?

S3: Build a tower of 14 cubes and a tower of 4 cubes

Sherwood: Should you have 2 towers if you are building an improper fraction?

In addition, when I asked, “Should you have 2 towers if you are building an improper fraction?” I should have asked the other students what their thoughts were about this representation. The problem is the time limitations for this in depth process. My goal is to continue developing my students’ mathematical reasoning within the time constraints allowed by my structured curriculum.

S3: Oh no, only one!

Sherwood: Do others agree with S3?

Students: Yes

S3: You have to do something with this fourth (referring to the stack of 4 cubes).

S3 then added the 4 cubes to the 14 cubes.

Sherwood: Does that represent 14/4?

S3: Oh no, that would be 18.

S2: I have a stack of 14.

Sherwood: Why do you have one stack of 14?

S2: Because its 14/4. 14 is the numerator and I think you have to do something with that.

Sherwood: What does this one cube represent?

Students: Some said 1/14 and others said ¼.

Sherwood: We are trying to represent 14/4

S1: I have an explanation here. This is what I think, I may not be correct but this is what I think. I don't have enough cubes for this. But if it is 14/4, one of these (a stack of 14) equals ¼.

S3: But it needs to be one stack.

S1: If you have 14 then you need another 14 and another 14.

S3: But it needs to be one stack.

S1: It is all very confusing.

Sherwood: Let's look back to a mixed number we made - 2 2/4. We would put 4 cubes together to represent a whole and another 4 cubes together to represent the other whole.

Students: That would represent the whole number 2.

Sherwood: What would each cube represent?

Students: ¼

Sherwood: The cubes represent what?

Students: The fraction.

Sherwood: What part of the fraction?

S1: The denominator.

Sherwood: So how can you represent 14/4?

S1: So one big thing of 14.

Students: Ohhhh!

S1: One cube represents ¼, the denominator. And if you have 14 one fourths, then you will have one stack of 14.

Sherwood: Do you guys agree or disagree with S1?

Students: Agree

Sherwood: So I hear you saying, is that the cubes represent the mixed number or the numerator, while the cubes individually represent the value of the denominator. Ok, let's try another one. Build 17/5. I gave students time, then asked them to explain what they drew.

My students had a very difficult time coming up with how to first make an improper fraction. I thought this would be very easy, but my assumption was wrong.

S2: I have a stack of 17.

Sherwood: Explain why.

S2: I have a stack of 17 because this shows 17 fifths. One cube represents 1/5. This is why I have 17. It is 17/5.

Sherwood: Please draw and label 17/5.

Students: Drew and labeled their improper fraction.

Sherwood: Is this more than a whole? If so, please prove it.

S3: Yes, the numerator is larger than the denominator. If the numerator is the same as the denominator then it equals one whole.

(Students were given some time)

S2: Since 5 is the denominator, I broke it into as many 5's as I could, which is 3. And I have 2 left over. Since 5 is the denominator it takes 5 fifths to make a whole. I could make 3 wholes and 2/5 were left over.

Sherwood: What type of number did you just describe?

S2: A mixed-number.

We did a few more examples.

Sherwood: How could we figure out how to convert between an improper fraction and a mixed number without building, drawing, and labeling?

S1: If you have 11/3 you can't divide 11 by 3 so we do 3 times what equals 11. 3x3 = 9 so then we subtract 9 from 11 so that is 2. Tthe whole number is 3, the numerator is 2, and the denominator is 3.

Sherwood: Can anyone explain how to do this for any number? What is the procedure for converting from improper fractions to mixed numbers?

S2: It is sort of like division. It is what S1's prediction was. You have to divide and then you subtract.

Sherwood: What do you mean?

S2: Well if you had the fraction 18/5, you are trying to get 5 into 18 as many times as you can. So 5 will go into 18, 3 times. And you have 3 left over. 5x3 is 15 and to get to 18 you have to add 3 more or you can take 18-15 which is 3.

Now many students were able to fully explain their reasoning behind the procedures they completed/did.

As I reflect upon by teaching practices of this research, I believe there are areas I could have improved upon. For instance, I would have asked the students how we could have tested S1 prediction instead of directing the students with my own agenda. As a teacher, it is important to listen to the students' ideas and guide the discussion based on their thinking. I directed the students in the direction I wanted them to go verses letting the students lead their own discussion. In addition, when I asked, "Should you have 2 towers if you are building an improper fraction?" I should have asked the other students what their thoughts were about this representation. The problem is the time limitations for this in depth process. My goal is to continue developing my students' mathematical reasoning within the time constraints allowed by my structured curriculum.

5. Conclusions and Limitations:

In conclusion, from analyzing this data my students were able to gain a conceptual understanding of how to convert between mixed numbers and improper fractions. From having students use manipulatives to build, draw, and label their fractions, they gained a greater understanding of why the procedures of converting between mixed numbers and improper fractions work. Through their dialogue, I observed how their mathematical reasoning was improving. By having these students work through these problems together, they learned how to express themselves logically so their peers could follow and understand their thought process. As a result, these students were able to learn a great deal from each other.

There are also limitations to this study. I was only able to work with three students because I had to get permission from their parents. In addition, we had limited time since we could only work on these lessons during our lunch period. By working with such a small group, I believe I had more of an impact. I am wondering if I would have achieved the same results if I were to conduct these same activities with the entire class. Since my time with these students was shorter than a class period, I was not able to expose the children to all three models of representing fractions. We used the unifix cubes the majority of the time. In the future, I would like to focus more on using multiple models of fractions since this concept is emphasized in the "Using Virtual Manipulatives to Model Computation with Fractions" and "Number, Operation, and Quantitative Reasoning" articles.

The "From the Classroom: Mixed Numbers Made Easy: Building and Converting Mixed Numbers and Improper Fractions" article's conceptual activities worked really well for my students. The one area they had trouble with was creating the improper fractions. I could have had the students label the cubes to help them see what one cube represents. In my specific case, I was unable to follow the recommended procedures as summarized in the "Reflecting on Learning Fractions without Understanding" article. I was not able to teach these students conceptual knowledge prior to procedural knowledge since the pre-test indicated that my students already had prior procedural knowledge.

6. <u>Next Steps:</u>

The result of my research demonstrates that these children understand how to convert between mixed numbers and improper fractions conceptually. However, I am wondering if these children will be able to better remember this information toward the end of the year. Many of my students understand how to switch from mixed numbers to improper fractions during the unit and when it comes time for the summative assessment. However, when we begin reviewing at the end of the year for the EOG's, many of the students forget the procedures.

I am wondering if I really focus on building this conceptual knowledge during the unit, before teaching the procedure, will more of my students remember it at the end of the year. I think they will based on the articles I have read and the research that has been done.

In addition, I could have taught a group of students this concept procedurally. Then I could test the two groups of students to determine their performance. These results would help me prove if students maintain the concepts better if they learn it conceptually verse procedurally only.

Through reading many researched based articles, there are many other models one can use to help students conceptually grasp these concepts of converting between mixed numbers and improper fractions. I believe students need to be exposed to a variety of representations since some students may understand a certain model more than another. The articles provided a variety of questions and activities I can implement in my classroom to facilitate conceptual understanding. I would like to implement more of these models into my instruction when teaching this concept to my entire class. I am curious to see which model students prefer to work with and which model makes it easier for my students to understand the concepts.

Based on this course and my research, I will attempt to incorporate the concepts of teaching conceptually and with reason into my lessons whenever possible. I will expose my students to many manipulatives and representations to assist my students in a deeper understanding of the concepts being taught.

This should improve my students' conceptual understanding and allow them to remember and maintain these concepts.

Resources

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